3-D SPH scheme with variable resolution: assessment of the optimal splitting refinement pattern

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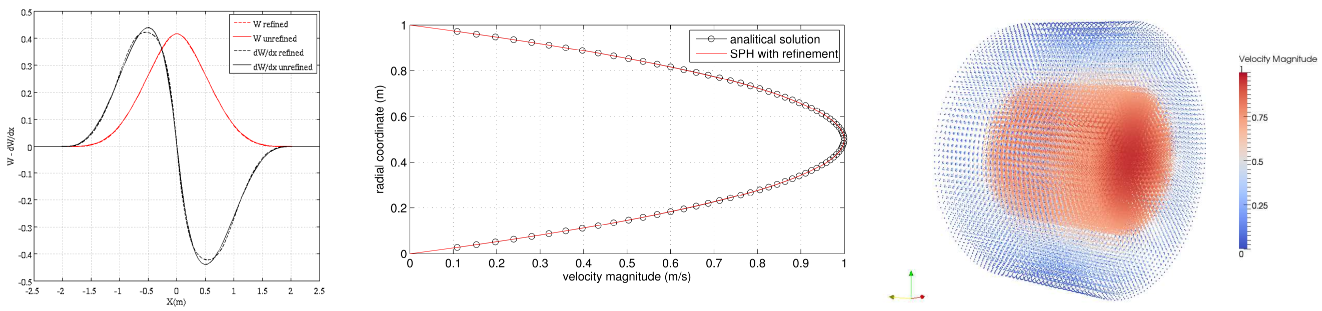
We present a variable resolution 3-D Smoothed Particle Hydrodynamics (SPH) algorithm for the Navier-Stokes equations. Previous works in SPH have achieved variable resolution by either remeshing, and particle insertion/removal techniques [1, 2], dynamically varying particle characteristics [5], or through variable smoothing lengths with dynamic particle splitting and coalescing according to pre-defined criteria [3,4]. Only recently have schemes appeared that offer both runtime particle splitting and coalescing (REF) to provide dynamic adaptive resolution. However all those previous works are developed considering 2-D schemes. The aim of this work is to develop a 3-D particle refinement scheme that includes both dynamic particle splitting and coalescing that minimises error within a SPH simulation. Unlike 2-D approaches, unique challenges exist to extend these ideas to 3-D.

Feldman and Bonet [6] defined the refinement stencil that should be used for 2-D models, but little attention has been dedicated to 3-D. In this work the global density error minimization algorithm has been extended to 3-D to define the position, smoothing length and other physical quantities of each daughter particles. In order to reduce the degrees of freedom in the splitting, the analysis has been conducted considering splitting patterns with spherical symmetry, in particular the following patterns have been analysed: cubic (8 particles), cubic with additional particles located in the centre of the faces, icosahedron and dodecahedron. In this way the relative particle position and the smoothing length of the daughter particles can be defined using only two parameters prior to a simulation to determine the optimum particle masses. An accuracy analysis of how the daughter particle positions and smoothing lengths affect the global density error and the mass distribution has been conducted for each refinement pattern. Finally the optimal refinement pattern and new smoothing lengths for 3-D simulations are identified. In order to conduct a thorough investigation different kernels have been used to demonstrate that the analysis is independent of the choice of kernel.

Figure 1-a shows the difference between the kernel and kernel gradient approximation for an unsplit particle (unrefined) and its daughter particles arrangement (refined) for the optimal 3-D refinement pattern. The error introduced is minimal.

In the weakly compressible numerical scheme a particle coalescing procedure is inserted, and the formulation adopted is variationally derived to ensure mass and momentum conservation also in presence of particles with different smoothing lengths. Furthermore, an improved particle shifting procedure (which can be adopted both for free surface and internal flows) is used, and its effects on the accuracy of the numerical scheme are also investigated. Figure 1-b shows the velocity profile for the 3-D Poseuille flow test case obtained with continuous particle splitting and coalescing and Reynold number Re=100. The agreement with the analytical solution is satisfactory. For the same test case the 3-D particle distribution is showed in Figure 1-c: the resolution is higher in the centre of the domain. More complex test cases,with free-surface and internal flows will be showed in the full paper in order to assess the accuracy of the scheme and the speedup obtained in comparison with an uniform resolution scheme.



(a) (b) (c)

Fig 1: the kernel and kernel gradient approximation for an unsplit particle (unrefined) and its daughter particles arrangement (refined) (a); Poseuille flow, velocity profile (b) and particle distribution in 3D (c)

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